



RESEARCH ARTICLE

DYNAMIC AND STABILITY ANALYSIS OF AN INCLINED BEAM SUBJECTED TO MOVING CONCENTRATED LOAD

Vincent Ifeanyi Ike

Civil Engineering, Faculty of Engineering, Nigeria Maritime University, Warri, Nigeria
Corresponding Author Email: ike.vincente@gmail.com

This is an open access journal distributed under the Creative Commons Attribution License CC BY 4.0, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

ARTICLE DETAILS

Article History:

Received 21 May 2023
Revised 24 June 2023
Accepted 28 July 2023
Available online 07 August 2023

ABSTRACT

The aim of this paper is to develop the dynamical and stability analysis of an inclined beam subjected to a moving concentrated load. The problem is solved for using the method of Laplace transformation for an initial-boundary-value problem, such that an integro-differential solution is obtained and used to simply deal with the condition of singularity by the load functional. The stability of general motion of the elastic system is determined by a direct variational approach. The result derived showed good agreement with that reported in literature.

KEYWORDS

Dynamic analysis, stability analysis, inclined beam, moving load, integro-differential problem

1. INTRODUCTION

A large margin of works in literatures focuses on moving loads on horizontal bridges and foundations to mention a few (Stanisic and Hardin, 1969; Tan, and Shore, 1968b; Sadiku, and Leipholz, 1987). It may be arguably stated that the intuition and perspective of moving load models on inclined structures elucidated in recent years to be a matter of concern in structural dynamics and stability. The growing complexity of mounting infrastructures, such as rails, bridges, and pipelines on uneven terrain to be subjected to moving load makes this research domain highly valid for engineers and mathematicians that are interested in formulating computational structural and load model and developing effective solutions. A good report on the application of moving load on inclined structures is presented in and hence, research becomes a basic modality to idealize our understanding on the performance mechanism of the system (Timan, 2015). As presented the dynamic response of an inclined beam with attention on centrifugal and coriolis force (Wu, 2005). Thereafter presented a nonlinear dynamic response of a beam given the effect of transverse shear deformation of the beam (Mamandi and Kargarnovin, 2011). To considered the dynamic and stability of the inclined beam in the context of an axially compressed load using the finite element method (Yang and Wang, 2019). In, they further provided insights onto the axial load effect on the beam stiffness based on a semi-analytical solution (Yang et al., 2020).

This paper tends to profer a simplistic integro-differential approach in and a direct variation method to solve for the inclined Euler beam (Sadiku and Leipholz, 1987). The approach takes advantage of the so-called extended Galerkin's method to highlight the intrinsic property of the inclined beam while applying the transform method to reduce the physical system onto a green's functional in the context of the moving concentrated load components.

2. PROBLEM STATEMENT

The following differential equation, and the initial and boundary conditions govern the flexural motion of the inclined beam (Yang and

Wang, 2019; Yang et al., 2020).

$$Dw'''' + P_a [1 - H(x - vt)]w'' + c\dot{w} + \mu\ddot{w} = (P_t + P_a w')\delta(x - vt) = P\delta(x - vt) \quad (1)$$

$$w(x, 0) = \dot{w}(x, 0) = 0, \quad (2)$$

$$w(0, t) = w'(0, t) = w(l, t) = w'(l, t) = 0.$$

The inclined beam having a moving concentrated load, P_0 is presented in Fig 1, m denotes the mass per unit length, E the Young's modulus, I the second moment of area, c the damping coefficient, w is the transverse deflection with respect to the inclined beam, the loads: P_t and P_a are the transverse and axial load component along the beam.

By Laplace transformation

$$\bar{w} = \int_0^\infty e^{-st} w(x, t) dt, \quad \bar{K} = \int_0^\infty e^{-st} K(x, t) dt \quad (3)$$

we get

$$D\bar{w}'''' + P_a [1 - H(x - vt)]\bar{w}'' + cs\bar{w} + \mu s^2 \bar{w} = \bar{K} \quad (4)$$

observing that $c = 2\omega_i \mu \xi_i$ represent the i^{th} modal damping ratio of the vibrating system

$$D\bar{w}'''' + P_a [1 - H(x - vt)]\bar{w}'' + 2\omega_i \mu \xi_i s \bar{w} + \mu s^2 \bar{w} = (\bar{P}_t + \bar{P}_a w')\delta(x - vt) \quad (5)$$

In order to obtain \bar{w} from (5), let the following series expansions be used

$$\bar{w}(s, x) = \sum_i A_i(s) W_i(x), \quad \bar{K}(s, x) = \sum_i \mu B_i(s) W_i(x) \quad (6)$$

Quick Response Code



Access this article online

Website:

www.actamechanicamalaysia.com

DOI:

10.26480/amm.02.2023.73.77

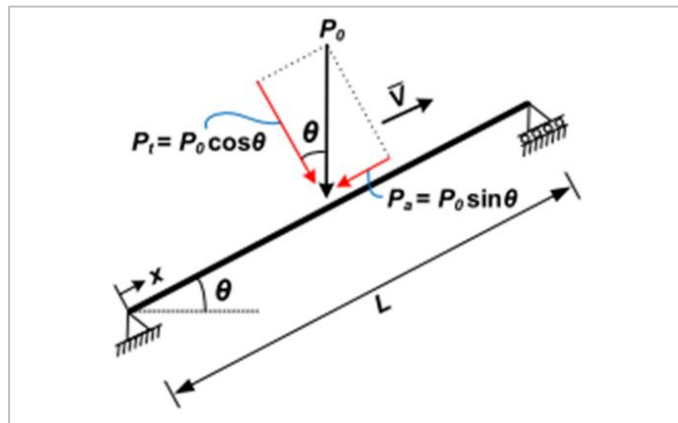


Figure 1: Components of moving load on inclined beam. Adopted from (Yang and Wang, 2019)

2.1 Eigenvalues and Eigenvectors

The coordinate functions are chosen as the eigenfunctions of the self-adjoint auxiliary problem as

$$DW_i^{(4)} + P_a [1 - H(x - vt)] W_i'' - \mu \omega_i^2 W_i = 0 \quad (7)$$

$$U[W_i(x)]_B = 0 \quad (8)$$

It follows simply from (7-8) that the function in

$$W_i(x) = \sum_{j=1}^{\infty} \alpha_{ij} Y_j(x) = \sum_{j=1}^{\infty} \alpha_{ij} \sin \frac{j\pi x}{l} \quad (9)$$

The function $Y_j(x)$ thus satisfy the boundary conditions. The solution maybe expressed in the so-called extended Galerkin's approach, and observing orthogonality condition so that we arrive at

$$\sum_{j=1}^{\infty} \alpha_{ij} \left\{ D \left(\frac{j\pi}{l} \right)^4 + P_a \left(\frac{j\pi}{l} \right)^2 \left[\frac{1}{2j\pi} \left(\sin \frac{2j\pi vt}{l} - \sin 2j\pi \right) - \frac{vt}{l} \right] - \mu \omega_i^2 \right\} \delta_{jk} + P_a \left(\frac{j\pi}{l} \right)^2 \left[\frac{2 \sin \pi(j-k) + 2 \sin \pi vt/l(j-k)}{2\pi(j-k)} - \frac{2 \sin \pi(j+k) + 2 \sin \pi vt/l(j+k)}{2\pi(j+k)} \right] \delta_{jk} = 0 \quad (10)$$

$$H(x - vt) = \begin{cases} 0 & \text{for } x \leq vt \\ 1 & \text{for } x > vt \end{cases} \quad (11)$$

noting that

$$\Pi_{jk} = \left[D \left(\frac{j\pi}{l} \right)^4 + P_a \left(\frac{j\pi}{l} \right)^2 \left[\frac{1}{2j\pi} \left(\sin \frac{2j\pi vt}{l} - \sin 2j\pi \right) - \frac{vt}{l} \right] - \mu \omega_i^2 \right] \delta_{jk} + P_a \left(\frac{j\pi}{l} \right)^2 \left[\frac{2 \sin \pi(j-k) + 2 \sin \pi vt/l(j-k)}{2\pi(j-k)} - \frac{2 \sin \pi(j+k) + 2 \sin \pi vt/l(j+k)}{2\pi(j+k)} \right] \delta_{jk} = 0 \quad (12)$$

The eigenvalues for the problem may be solved for if we attempt to find the determinant of

$$\Pi_{jk} \det[\Pi_{jk}] = 0 \quad (13)$$

By back substitution of eigenvalues in (10), the coefficients are obtained. We see from (7-8) that the coordinate functions satisfy the following orthonormality conditions; assuming that the weight of the moving mass is far negligible when compared to that of the beam

$$\int_0^l \mu Y_j(\phi) Y_k(\phi) dx = \delta_{jk} \quad (14)$$

One must realize that the nature of operator A in (6) may result to an expression for a complicated integro-differential equation. Therefore, in order to keep things simple here we set (6) in (5) as

$$\sum_i A_i \{ DW_i^{(4)} + P_a [1 - H(x - vt)] W_i'' + \mu s^2 W_i \} = \sum_i \mu (B_i - 2\omega \xi s) W_i \quad (15)$$

Taking note of (7) and (15), we get

$$\sum_i A_i \mu (\omega_i^2 + s^2) W_i = \sum_i \mu (B_i - 2\omega \xi s) W_i \rightarrow A_i = \frac{B_i - 2\omega \xi s}{\omega_i^2 + s^2} \quad (16)$$

From (6a)

$$\bar{w}(x, s) = \sum_i \frac{B_i - 2\omega \xi s}{\omega_i^2 + s^2} W_i(x) \quad (17)$$

but from (6b), it is easily deduced that

$$B_i = \int_0^l \bar{P}(\eta, s) W_i(\eta) d\eta \quad (18)$$

So that

$$\bar{w}(x, s) = \sum_i \frac{W_i(x)}{\omega_i^2 + s^2} \int_0^l [\bar{P}(\eta, s) - 2\omega \xi s] W_i(\eta) d\eta \quad (19)$$

By inversion, the Laplacian transform the solution becomes

$$w(x, t) = \sum_i \int_0^t \int_0^l \frac{W_i(x) W_i(\eta)}{\omega_i} \sin[\omega_i(t - \tau)] P(\eta, \tau) d\eta d\tau - 2\xi \sum_i \int_0^l W_i(x) W_i(\eta) \cos(\omega_i t) d\eta \quad (20)$$

The solution turns actual if the convergence of the series involved in (20) can be proved. An integration by parts with respect to t is carried out once on the right hand side, and the following is obtained:

$$w(x, t) = \sum_i \int_0^t W_i(x) W_i(\eta) \left\{ \frac{P(\eta, t)}{\omega_i^2} - \frac{P(\eta, 0)}{\omega_i^2} \cos \omega_i t - \int_0^t \frac{1}{\omega_i^2} \cos \omega_i(t - \tau) \frac{\partial P}{\partial \tau} d\tau - 2\xi \cos(\omega_i t) \right\} d\eta \quad (21)$$

Now, let the third term on the right side of (21) be investigated on the successively real eigenvalues

$$\left| \sum_i \int_0^t \int_0^l \frac{W_i(x) W_i(\eta)}{\omega_i^2} \cos \omega_i(t - \tau) \frac{\partial P}{\partial \tau} d\tau d\eta \right| \leq \sum_i \int_0^t \int_0^l \left| \frac{W_i(x) W_i(\eta)}{\omega_i^2} \right| |\cos \omega_i(t - \tau)| \left| \frac{\partial P}{\partial \tau} \right| d\tau d\eta \leq \int_0^t \left| \frac{\partial P}{\partial \tau} \right| d\tau \sum_i \int_0^l \left| \frac{W_i(x) W_i(\eta)}{\omega_i^2} \right| d\eta \quad (22)$$

Assuming that eigenvalues of the auxiliary problem (7) are uniformly bounded in $0 \leq x \leq l$

$$\sum_i \int_0^l \left| \frac{W_i(x) W_i(\eta)}{\omega_i^2} \right| d\eta \leq l W^2 \sum_i \frac{1}{\omega_i^2} \leq \infty. \quad (23)$$

In the case of the last term on the right side in (20); since $w(x, 0) = 0$ is assumed to satisfy the boundary conditions in (2), one can expand in terms of the eigenfunctions according to Hilbert's expansion theorem as

$$\begin{aligned} & \sum_i \left\{ \int_0^l 2\xi W_i(\eta) \cos(\omega_i t) w(\eta, t) d\eta \right\} W_i(x) \\ & \leq \sum_i \left\{ \left| \int_0^l 2\xi W_i(\eta) w(\eta, t) d\eta \right| |\cos(\omega_i t)| \right\} W_i(x) \\ & \leq \sum_i \left\{ \left| \int_0^l 2\xi W_i(\eta) w(\eta, t) d\eta \right| \right\} |W_i(x)| < T(x, t). \end{aligned} \quad (24)$$

In reference to a control functional, $T(x, t)$ in (24) is obviously bounded as long as $P_a < P_{a, \text{crit}}$, i.e., as long as the eigenvalues are real quantities. Upon introducing the forcing function as

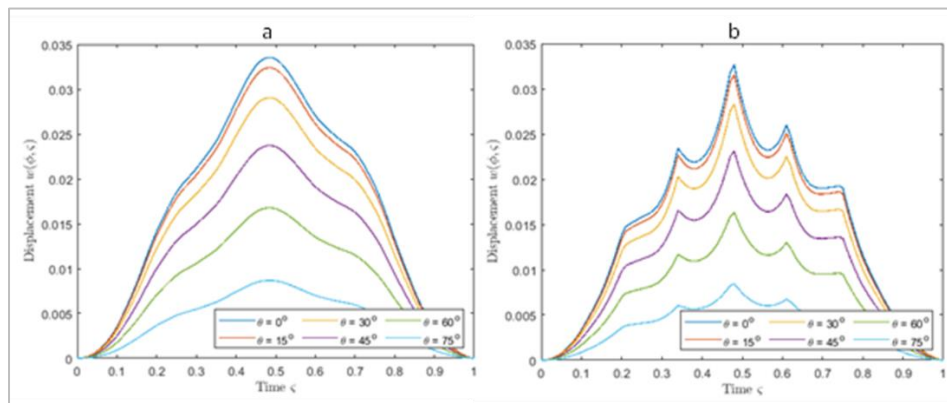


Figure 3: Dynamic response of undamped(a) and damped(b) beam at $v = 10.6988$ m/sec

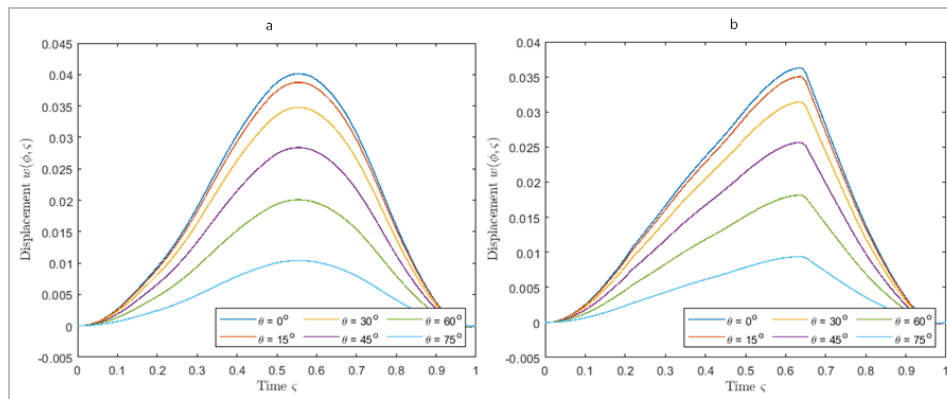


Figure 4: Dynamic response of undamped(a) and damped(b) beam at $v = 33.8327$ m/sec

Table 1: Maximum deflection w_{max} (in 10^{-2}) of inclined beam with different speeds

Beam Inclination	$v = 3.3833$ [S=0.002]		$v = 5.3494$ [S=0.005]		$v = 10.6988$ [S=0.02]		$v = 33.8327$ [S=0.2]	
	Undamped	Damped	Undamped	Damped	Undamped	Damped	Undamped	Damped
0°	3.19	2.94[3.14]	3.23	3.35	3.35	3.27[3.18]	4.01	3.63[3.37]
15°	3.08	2.84[3.09]	3.12	3.24	3.24	3.16[3.13]	3.88	3.50[3.32]
30°	2.76	2.55[2.82]	2.80	2.90	2.90	2.83[2.86]	3.48	3.14[3.03]
45°	2.26	2.08[2.34]	2.29	2.37	2.37	2.31[2.37]	2.84	2.57[2.51]
60°	1.59	1.47[1.68]	1.62	1.68	1.68	1.63[1.69]	2.01	1.81[1.80]
75°	0.825	0.761[0.876]	0.837	0.868	0.868	0.846[0.883]	1.04	0.939[0.938]

The values in bracket is that reported in [7].

Table 2: Minimum Buckling Load at Different Angles of Inclination

θ	\bar{P}_{cr}	λ ($\xi = 0$)	λ ($\xi = 0.02$)
0°	∞	0, 19.7392	0.0066, 19.7326, 19.7458
30°	19.7392		
60°	11.3964		
80°	10.0219		
89°	9.8711 [10.45]		
90°	9.8696		

The values in bracket is that reported in (Yang and Wang, 2019).

It is interesting to observe that the minimum critical loads liable to trigger buckling at any inclination in this approach are reasonably below the critical loads discussed in (Yang and Wang, 2019). The concept of direct variational approach provides a rather exact load capable of immediate distortion of the dynamic system compared to the reference herewith to test the stability. The result agreed that as the inclination reduces, a higher load magnitude will be required to reach buckling in the system. It also revealed that the major stability problem of the dynamic system is

conditioned by the magnitude of the axial force component.

4. CONCLUSION

A close form solution for the moving load of transverse and axial components is dealt with. The resolve by integro-differential approach presented a rather simplistic view in understanding the performance mechanism of the system when compared to other reports. Some findings

were highlighted with the aid of examples. The solutions following showed that the magnitude of deflection in beam reduces as the angle of beam inclination increase. The drop in deflection of the beam results in a corresponding decrease in the buckling load required to provide stability of the beam.

REFERENCES

Mamandi, A., and Kargarnovin, M.H., 2011. Dynamic analysis of an inclined timoshenko beam traveled by successive moving masses/forces with inclusion of geometric non-linearities. Acta Mechanica, vol. 218, Pp. 9 – 29, <https://doi.org/10.3233/SAV-2010-0610>

Sadiku, S., and Leipholz, H.H.E., 1987. On the dynamics of elastic systems with moving concentrated masses." Ingenieur Archiv, vol. 57, Pp. 223-242, <https://doi.org/10.1007/bf02570609>

Stanisic, M.M., and Hardin, J.C., 1969. On response of beams to an arbitrary

- number of moving masses. *Journal of the Franklin Institute*, vol. 287, no. 2, Pp. 115-123, 1969. [https://doi.org/10.1016/0016-0032\(69\)90120-3](https://doi.org/10.1016/0016-0032(69)90120-3)
- Tan, C.P., and Shore, S.S., 1968b. Response of a horizontally curved bridge to moving load. *Journal of the Structural Division*, vol. 94, ST9, Pp. 2135-2151, <https://doi.org/10.1061/jsdeag.0002068>
- Timan, P.E., 2015. Why monorail systems provide a great solution for metropolitan areas. *Urban Rail Transit*, vol. 1, Pp. 13 - 25, <https://doi.org/10.1007/s40864-015-0001-1>
- Wu, J.J., 2005. Dynamic analysis of an inclined beam due to moving loads. *Journal of Sound and Vibration*, vol. 288, Pp. 107-131. <https://doi.org/10.1016/j.jsv.2004.12.020>
- Yang, D.S., and Wang, C.M., 2019. Dynamic response and stability of an inclined Euler beam under a moving vertical concentrated load. *Engineering Structures*, vol. 186, Pp. 243-254, <https://doi.org/10.1016/j.engstruct.2019.01.140>
- Yang, D.S., Wang, C.M., and Pan, W.H., 2020. Further insights into moving load problem on inclined beam based on semi-analytical solution. *Structures*, vol. 26, Pp. 247 - 256, <https://doi.org/10.1016/j.istruc.2020.03.050>

