



RESEARCH ARTICLE

BENDING ANALYSIS OF FG POROUS BEAM BASED ON SIMPLE TIMOSHENKO BEAM ELEMENTS

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ABSTRACT

In this document, the static bending of functionally graded porous (FGP) beam is studied under various boundary conditions and uniform load. Besides, finite element procedure is based on the simple Timoshenko beam theory. The effects of material properties like two types of porosity on bending behaviors are also considered. The solutions achieved in this document are given and evaluated with other solutions in the cited papers to check the feasibility in deployment the formula and creating the Matlab program. Furthermore, this document helps researchers to get some information about the bending behaviors of the proposed structures.

KEYWORDS

Static bending, Rotation, Transverse displacement, FGP beam, Simple Timoshenko beam.

1. INTRODUCTION

In this era, functionally graded (FG) material has become one of the advanced materials and it is applied in many different fields. It is commonly made from a mixture of ceramic and metal and provided the continuous variation of material properties from the top surface to the bottom surface of structure. For instance, in the defense industry, nuclear tanks, spacecraft, etc. are produced based on the above material (Almasi et al., 2016; Singh et al., 2021; Naebe and Shirvanimoghaddam, 2016; Singh and Rastogi, 2021). Due to the high applicability of functionally graded material, many studies related to various theories have been presented to comment the mechanical behavior of FG structures as (Beg and Yasin, 2021; Chen et al., 2015; Hoang, 2020; Hoang et al., 2021; Hoang et al., 2020; Ton-That et al., 2021). However, porosity of the material can occur during the manufacturing process (Cho et al., 2020; Martínez et al., 2020; Xiong et al., 2021).

So, an investigation related to this topic should be considered as soon as possible to have a good knowledge of porosity effect on mechanical behavior of FG structures. Based on three types of structure such as beam, plate and shell, researchers are usually interested in beam structures because of its wide actualizations. Furthermore, many individual beam theories were applied to calculate beam structures such as simple beam theory, classical beam theory, first-order shear deformation theory or third-order shear deformation theory (Katili, et al., 2020; Li, et al., 2013; Şimşek et al., 2013; Li, 2008; Wu et al., 2018; Yin et al., 2021; Singiresu, 2018; Avhad and Sayyad, 2020; Kadoli et al., 2008; Ton-That, 2020). As expected, the simple Timoshenko beam model helps us to decrease the computational cost with the allowable error of results. Moreover, beams created by FG porous materials must be investigated as much as possible to guide the designer have right knowledge about the mechanical properties.

Some cited papers on static bending analysis of FG beams can be listed here. Paper gave a new way to study the FG beams without porosity and with the shear deformation and rotary inertia (Li, 2008). Avhad and

partner used higher order shear and normal deformation theory for static bending of functionally graded composite beams curved in elevation without porosity (Avhad and Sayyad, 2020). Governing equations in this paper were obtained using the principle of virtual work. Atmane *et al.* presented some behaviors of FG porous beams resting on elastic foundations such as bending, free vibration and buckling behavior via a well organized quasi-3D theory (Atmane, et al., 2017). Souhir et al. used a novel finite element beam model to present the impacts of porosity on bending behaviors of FG beams, etc (Zghal et al., 2020). Last but not least, this document is written to describe the bending behaviors of FG porous beams. The next three parts are shown in this document. Part 2 gives the deployments. Part 3 presents some solutions of FG porous beams. Finally, a few statements are written in Part 4 respectively.

2. DEPLOYMENTS

With length L , width b and thickness h , a beam based on FG porous material is now studied. By changing phases from ceramic to metal along the z direction, the parameter V_k of metal phase related to a power-law distribution can be depicted in

$$V_k = (0.5 + z/h)^n \quad (1)$$

$$V_k + V_g = 1 \quad (2)$$

where the power-law index is n .

Regarding the production activity, porosities may exist as an imperfection in the FG beams. There are even and uneven distributions of porosity as described in Figure 1. By using the modified rule of mixture in which the porosity volume fraction, α , $0 \leq \alpha < 1$, affects averagely the material volume fraction of each constituent, the effective material properties of the FGP beam are then determined

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$$P(z, \alpha) = (P_k - P_g)V_k + P_g - \frac{\alpha}{2}(P_k - P_g) \quad \text{even distribution} \quad (3)$$

$$P(z, \alpha) = (P_k - P_g)V_k + P_g - \frac{\alpha}{2}(P_k - P_g)(h - 2|z|)/h \quad \text{uneven distribution} \quad (4)$$

With even phase, the porosity is uniformly distributed along the cross section of beam and with uneven phase, the porosity nearly appears in the middle surface of the cross section and vanished in the top or bottom surfaces.

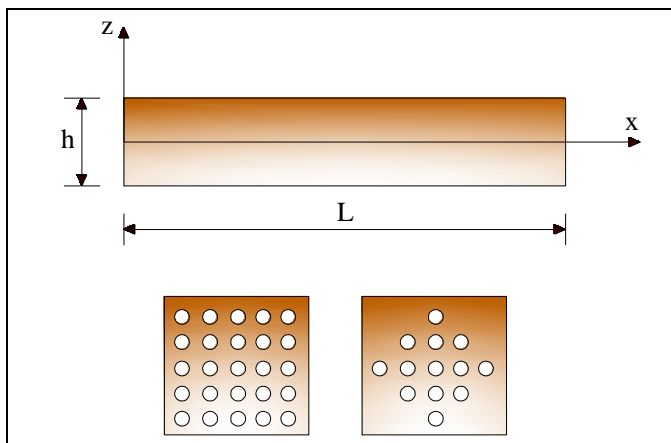


Figure 1: FGP beam and two distributions

Related to finite element analysis (FEA), the DOFs connected with a node of a simple Timoshenko beam element are w and φ as described in Figure 2. Besides, based on the simple Timoshenko beam theory, the stiffness matrix of element is formulated

$$S_{pt} = \frac{E_{pt} I_{pt}}{L_{pt}^3 (1 + \Gamma)} \begin{bmatrix} 12 & 6L_{pt} & -12 & 6L_{pt} \\ 6L_{pt} & (4 + \Phi)L_{pt}^2 & -6L_{pt} & (2 - \Phi)L_{pt}^2 \\ -12 & -6L_{pt} & 12 & -6L_{pt} \\ 6L_{pt} & (2 - \Phi)L_{pt}^2 & -6L_{pt} & (4 + \Phi)L_{pt}^2 \end{bmatrix} \quad (5)$$

with

$$\Gamma = \frac{12E_{pt} I_{pt}}{G_{pt} f_{sc} A_{pt} L_{pt}^2} \quad (6)$$

and the shear correct factor f_{sc} takes the value 5/6.

The equation of element can be written by using the principle of minimum total potential energy:

$$\frac{E_{pt} I_{pt}}{L_{pt}^3 (1 + \Gamma)} \begin{bmatrix} 12 & 6L_{pt} & -12 & 6L_{pt} \\ 6L_{pt} & (4 + \Phi)L_{pt}^2 & -6L_{pt} & (2 - \Phi)L_{pt}^2 \\ -12 & -6L_{pt} & 12 & -6L_{pt} \\ 6L_{pt} & (2 - \Phi)L_{pt}^2 & -6L_{pt} & (4 + \Phi)L_{pt}^2 \end{bmatrix} \begin{Bmatrix} w_i \\ \varphi_i \\ w_j \\ \varphi_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ m_i \\ f_j \\ m_j \end{Bmatrix} \quad (7)$$

After assembly, the transverse displacements and rotations at all nodes are achieved by:

$$\mathbf{u}_{full} = \mathbf{S}_{full}^{-1} \mathbf{F}_{full} \quad (8)$$

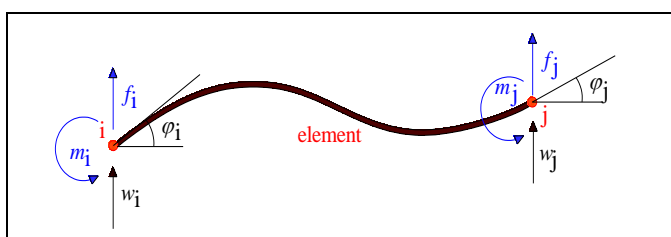


Figure 2: A Timoshenko beam element

Three letters 'F', 'S' and 'C' are used to aim the free, simply supported as well as clamped condition. The combined boundary conditions (BCs) are shown in:

$$(SS) \quad w(0) = w(L) = 0 \quad (9)$$

$$(CS) \quad w(0) = \varphi(0) = 0, w(L) = 0 \quad (10)$$

$$(CC) \quad w(0) = \varphi(0) = 0, w(L) = \varphi(L) = 0 \quad (11)$$

$$(CF) \quad w(0) = \varphi(0) = 0 \quad (12)$$

The finite element analysis is used in several steps:

- Step 1: Saving geometric sizes and material parameters.
- Step 2: Calculating constitutive matrix.
- Step 3: Calculating stiffness matrix and force vector for element.
- Step 4: Assembling all components.
- Step 5: Connecting BCs.
- Step 6: Solving equation of system for static bending.
- Step 7: Display w and φ at all nodes of system.

3. NUMERICAL SOLUTIONS

Firstly, the convergence of this model is verified in simply supported (SS) FGP beam under a uniform load $q = 10^6 \text{ N/m}^2$. The material properties of beam is made of (Al/Al₂O₃) composite with all details as in Table 1. At position $L/2$, the dimensionless transverse displacement is formulated by

$$\bar{w} = 100 \frac{E_k h^3}{q L^4} w\left(\frac{L}{2}\right). \quad \text{The values of normalized deflection for FGP beams}$$

with $L/h = 5$, three values of porosity coefficient $\alpha = 0, 0.1$ & 0.2 and power-law index $n = 2$ are given in Table 2 and compared with other results from another beam theories of (Atmane et al., 2017; Zghal et al., 2020).

Table 1: The input data

(Al / Al₂O₃)

$$E_k = 70 \times 10^9 \text{ Pa}, \nu_k = 0.3, \rho_k = 2702 \text{ kg/m}^3$$

$$E_g = 380 \times 10^9 \text{ Pa}, \nu_g = 0.3, \rho_g = 3960 \text{ kg/m}^3$$

Table 2: Solutions of \bar{w} for FG porous beams (SS) with $L/h = 5$ and $n = 2$

α	(Zghal et al., 2020)	(Atmane et al., 2017)	Present
0	5.20	5.35	5.35
0.1	5.82	6.22	6.06
0.2	6.63	7.38	6.97

The solutions achieved from the document are completely approximate with other solutions. The error among them may be explained by the different beam theories used in all cited papers. In addition, the proposed result for each case of α is within the range of two solutions of Atmane *et al.* related to the refined plate theory for functionally graded beams and Zghal *et al.* based on the novel model for finite element analysis. Secondly, the changing of $w = w(x = 0 \rightarrow L)$ and φ along the length of (CC) FG porous beams with $L/h = 5$, $\alpha = 0.2$ and two distributions of porosity are shown in Figures 3 & 4. When n increases, the values of w and φ also increase respectively. Moreover, the comparison of these values for two types of porosity is illustrated in Figure 5 & 6. Obviously the effect of even porosity outweighs that of uneven porosity.

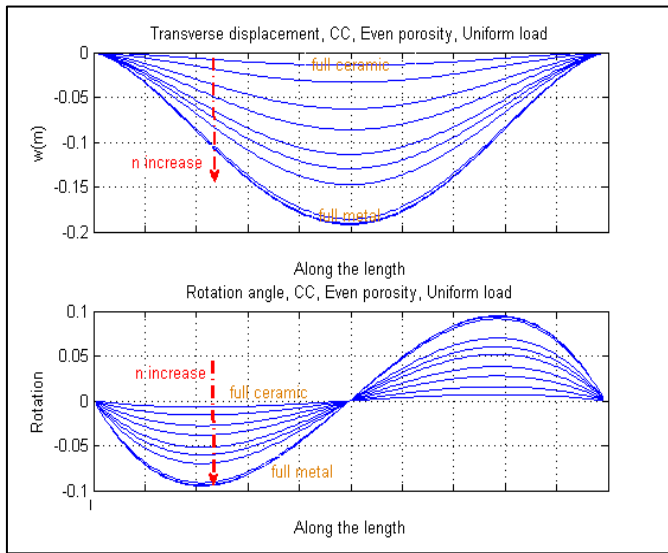


Figure 3: The variation of w and ϕ along the length of (CC) FG porous beams with $\alpha = 0.2$, even distribution and under uniform load

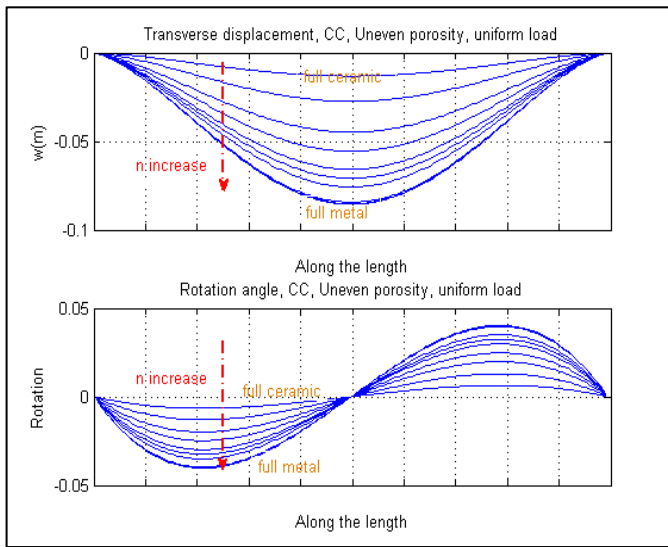


Figure 4: The variation of w and ϕ along the length of (CC) FG porous beams with $\alpha = 0.2$, uneven distribution and under uniform load

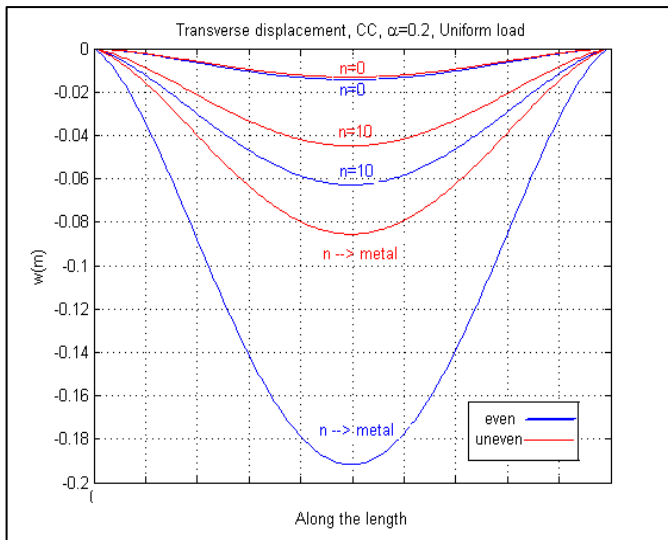


Figure 5: The comparison of transverse displacements of (CC) FGP beams for two types of porosity

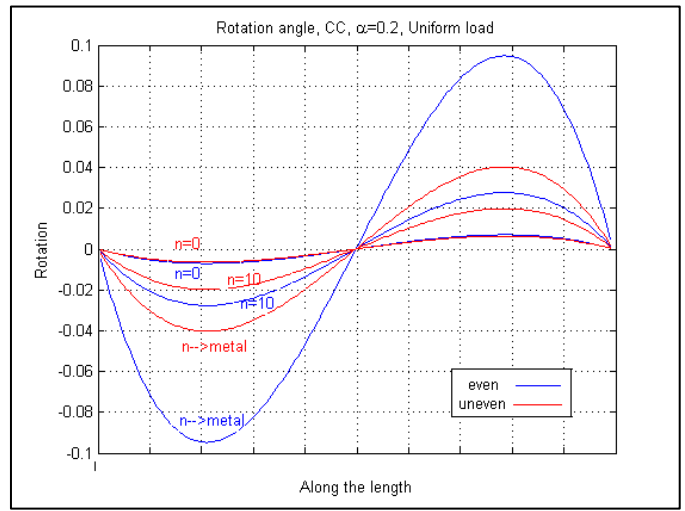


Figure 6: The comparison of rotations of (CC) FGP beams for two types of porosity

Thirdly, by changing the boundary condition from (CC) to (CS) and (CF), the bending results of FG porous beams can be seen in Figures 7-10. Once again, the effect of even porosity outweighs that of uneven porosity respectively.

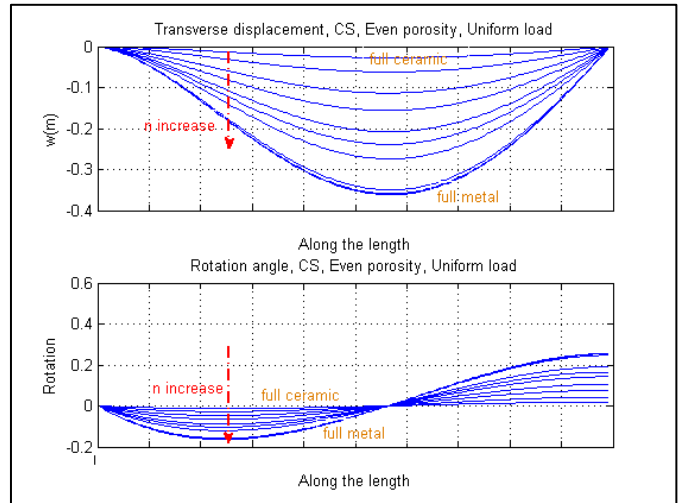


Figure 7: The variation of w and ϕ along the length of (CS) FG porous beams with $\alpha = 0.2$, even distribution and under uniform load

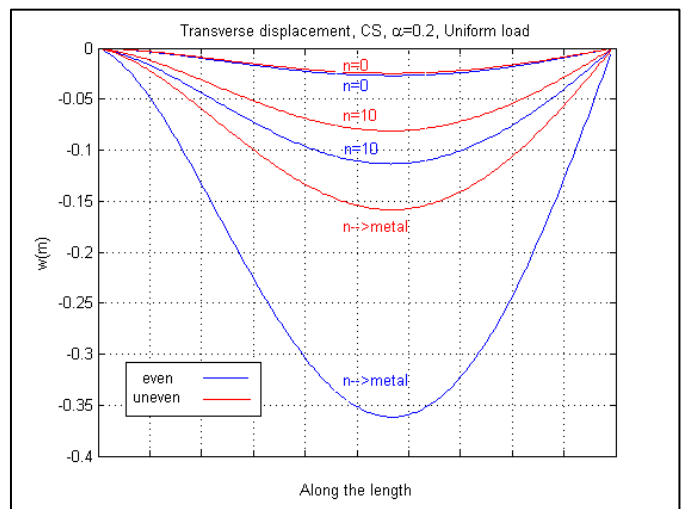


Figure 8: The comparison of transverse displacements of (CS) FGP beams for two types of porosity

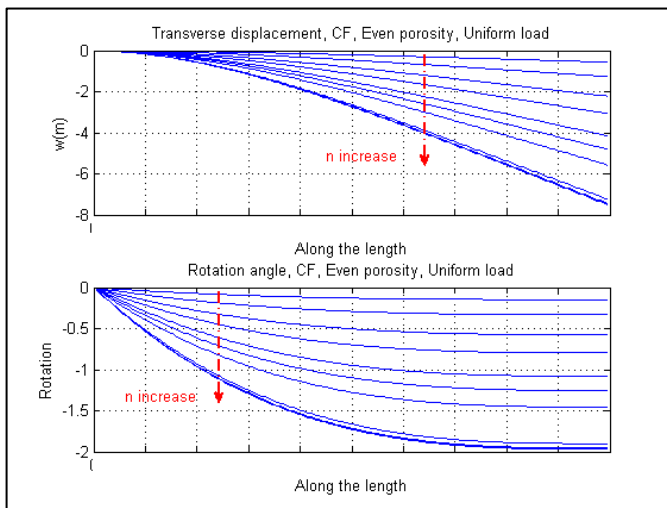


Figure 9: The variation of w and φ along the length of (CF) FG porous beams with $\alpha = 0.2$, even distribution and under uniform load

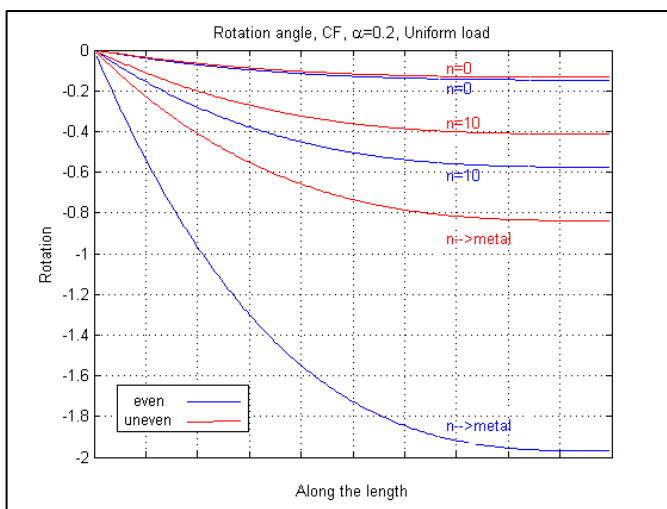


Figure 10: The comparison of rotations of (CF) FGP beams for two types of porosity.

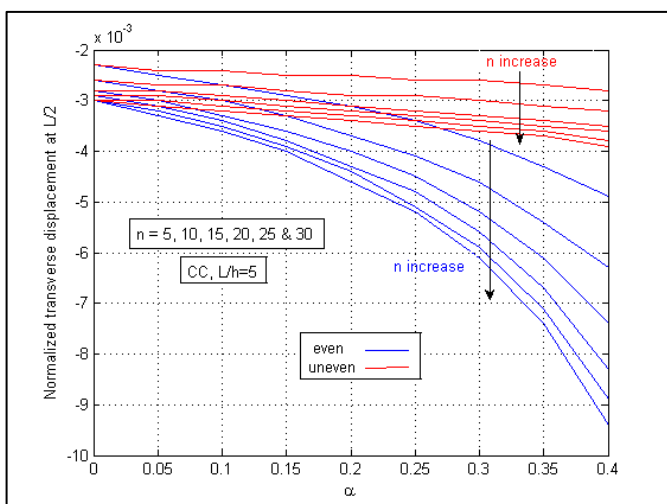


Figure 11: The \bar{w} of (CC) FG porous beams with many values of α , even/uneven distribution and $n = 5, 10, 15, 20, 25$ & 30

Finally, by varying the parameter α from 0 to 0.4, the solutions of the dimensionless transverse displacement $\bar{w} = \frac{1}{h} w\left(\frac{L}{2}\right)$ at position $L/2$ of FG porous beams with (CC) are depicted in Figure 11 for two distributions of porosity. The deflection of FGP beam will increase when the porosity value increases and this comment still exists for all values of power-law index.

4. CONCLUSION

The static bending analysis of FG porous beams under two distributions of porosity and four kinds of BC are presented. The solutions of this document are in good agreement with other solutions in cited papers. Although the work is not new, the main purpose is to affirm the feasibility of the simple Timoshenko beam theory to study the FG porous beams with acceptable solutions.

REFERENCES

- Almasi, D., Sadeghi, M., Lau, W.J., Roozbahani, F., Iqbal, N., 2016. Functionally graded polymeric materials: A brief review of current fabrication methods and introduction of a novel fabrication method. *Materials Science and Engineering: C*, 64, Pp. 102-107. doi: <https://doi.org/10.1016/j.msec.2016.03.053>
- Atmane, H.A., Tounsi, A., Bernard, F., 2017. Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations. *International Journal of Mechanics and Materials in Design*, 13 (1), Pp. 71-84. doi: 10.1007/s10999-015-9318-x
- Avhad, P.V., Sayyad, A.S., 2020. Static analysis of functionally graded composite beams curved in elevation using higher order shear and normal deformation theory. *Materials Today: Proceedings*, 21, Pp. 1195-1199. doi: <https://doi.org/10.1016/j.matpr.2020.01.069>
- Beg, M.S., Yasin, M.Y., 2021. Bending, free and forced vibration of functionally graded deep curved beams in thermal environment using an efficient layerwise theory. *Mechanics of Materials*, 159, Pp. 103919. doi: <https://doi.org/10.1016/j.mechmat.2021.103919>
- Chen, D., Yang, J., Kitipornchai, S., 2015. Elastic buckling and static bending of shear deformable functionally graded porous beam. *Composite Structures*, 133, Pp. 54-61. doi: <https://doi.org/10.1016/j.compstruct.2015.07.052>
- Cho, S., Kim, J., Lee, S.B., Choi, M., Kim, D.H., Jo, I., Kim, Y., 2020. Fabrication of functionally graded hydroxyapatite and structurally graded porous hydroxyapatite by using multi-walled carbon nanotubes. *Composites Part A: Applied Science and Manufacturing*, 139, Pp. 106138. doi: <https://doi.org/10.1016/j.compositesa.2020.106138>
- Dev Singh, D., Arjula, S., Raji Reddy, A., 2021. Functionally Graded Materials Manufactured by Direct Energy Deposition: A review. *Materials Today: Proceedings*. doi: <https://doi.org/10.1016/j.matpr.2021.04.536>
- Hoang, L.T.T., 2020. A Combined Strain Element to Functionally Graded Structures in Thermal Environment. *Acta Polytechnica*, 60 (6), Pp. 528-539.
- Hoang, L.T.T., Nguyenvan, H., 2021. A Combined Strain Element in Static, Frequency and Buckling Analyses of Laminated Composite Plates and Shells. *Periodica Polytechnica Civil Engineering*, 65 (1), Pp. 56-71. doi: 10.3311/PPci.16809
- Kadoli, R., Akhtar, K., Ganesan, N., 2008. Static analysis of functionally graded beams using higher order shear deformation theory. *Applied Mathematical Modelling*, 32 (12), Pp. 2509-2525. doi: <https://doi.org/10.1016/j.apm.2007.09.015>
- Katili, I., Sahril, T., Katili, A.M., 2020. Static and free vibration analysis of FGM beam based on unified and integrated of Timoshenko's theory. *Composite Structures*, 242, Pp. 112130. doi: <https://doi.org/10.1016/j.compstruct.2020.112130>
- Li, S.R., Cao, D.F., Wan, Z.Q., 2013. Bending solutions of FGM Timoshenko beams from those of the homogenous Euler-Bernoulli beams. *Applied Mathematical Modelling*, 37 (10), Pp. 7077-7085. doi: <https://doi.org/10.1016/j.apm.2013.02.047>
- Li, X.F., 2008. A unified approach for analyzing static and dynamic behaviors of functionally graded Timoshenko and Euler-Bernoulli beams. *Journal of Sound and Vibration*, 318 (4), Pp. 1210-1229. doi: <https://doi.org/10.1016/j.jsv.2008.04.056>
- Martínez, C., Guerra, C., Silva, D., Cubillos, M., Briones, F., Muñoz, L., Sancy, M., 2020. Effect of porosity on mechanical and electrochemical properties of Ti-6Al-4V alloy. *Electrochimica Acta*, 338, Pp. 135858. doi: <https://doi.org/10.1016/j.electacta.2020.135858>

- Naebe, M., Shirvanimoghaddam, K., 2016. Functionally graded materials: A review of fabrication and properties. *Applied Materials Today*, 5, Pp. 223-245. doi: <https://doi.org/10.1016/j.apmt.2016.10.001>
- Şimşek, M., Kocatürk, T., Akbaş, Ş.D., 2013. Static bending of a functionally graded microscale Timoshenko beam based on the modified couple stress theory. *Composite Structures*, 95, Pp. 740-747. doi: <https://doi.org/10.1016/j.compstruct.2012.08.036>
- Singh, R.K., Rastogi, V., 2021. A review on solid state fabrication methods and property characterization of functionally graded materials. *Materials Today: Proceedings*. doi: <https://doi.org/10.1016/j.matpr.2021.03.634>
- Singiresu, S.R., 2018. *The Finite Element Method in Engineering* (Sixth ed.): Elsevier
- Ton-That, H.L., 2020. The Linear and Nonlinear Bending Analyses of Functionally Graded Carbon Nanotube-Reinforced Composite Plates Based on the Novel Four-Node Quadrilateral Element. *European Journal of Computational Mechanics*, 29 (1), Pp. 139-172. <https://doi.org/10.13052/ejcm2642-2085.2915>
- Ton-That, H.L., 2020. Improvement on eight-node quadrilateral element (IQ8) using twice-interpolation strategy for linear elastic fracture mechanics. *Engineering Solid Mechanics*, 8 (4), Pp. 323-336. doi: [10.5267/j.esm.2020.3.005](https://doi.org/10.5267/j.esm.2020.3.005)
- Ton-That, H.L., Nguyenvan, H., Chau-Dinh, T., 2021. A novel quadrilateral element for analysis of functionally graded porous plates/shells reinforced by graphene platelets. *Archive of Applied Mechanics*, 91 (6), Pp. 2435-2466. doi: [10.1007/s00419-021-01893-6](https://doi.org/10.1007/s00419-021-01893-6)
- Wu, D., Gao, W., Hui, D., Gao, K., Li, K., 2018. Stochastic static analysis of Euler-Bernoulli type functionally graded structures. *Composites Part B: Engineering*, 134, Pp. 69-80. doi: <https://doi.org/10.1016/j.compositesb.2017.09.050>
- Xiong, Y., Han, Z., Qin, J., Dong, L., Zhang, H., Wang, Y., Li, X., 2021. Effects of porosity gradient pattern on mechanical performance of additive manufactured Ti-6Al-4V functionally graded porous structure. *Materials & Design*, 208, Pp. 109911. doi: <https://doi.org/10.1016/j.matdes.2021.109911>
- Yin, S., Xiao, Z., Deng, Y., Zhang, G., Liu, J., Gu, S., 2021. Isogeometric analysis of size-dependent Bernoulli-Euler beam based on a reformulated strain gradient elasticity theory. *Computers & Structures*, 253, Pp. 106577. doi: <https://doi.org/10.1016/j.compstruc.2021.106577>
- Zghal, S., Ataoui, D., Dammak, F., 2020. Static bending analysis of beams made of functionally graded porous materials. *Mechanics Based Design of Structures and Machines*, Pp. 1-18. doi: [10.1080/15397734.2020.1748053](https://doi.org/10.1080/15397734.2020.1748053)

