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COMPUTATIONAL INVESTIGATION OF MHD FREE CONVECTION ON A FLAT VERTICAL PLATE EMBEDDED WITH MICRO POLAR FLUID

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ABSTRACT

The theory of micropolar fluid has a wide range of applications in fluid mechanics and other related areas, in which coupling between the spin of each particle and microscopic velocity is taken into account. Using this theory many researchers have discussed the dynamics of micropolar fluid. The micropolar fluid flow equations constitute a coupled system of equations and are complicated. The aim of the present work is to study the free convection along a vertical plate heat and mass transfer in micropolar fluid in the presence of double stratification, MHD and suction/injection effects.

KEYWORDS

MHD, micro polar fluid, convection, vertical plate, injection effects

1. INTRODUCTION

Fluids treated in the classical theory of fluid mechanics and heat transfer are the ideal fluid and the Newtonian fluid. The former is completely frictionless, so that shear stress is absent, while the latter has a linear relationship between shear stress and shear rate. Unfortunately, the behavior of many real fluids used in the mechanical and chemical industries such as suspensions of particles and fibers, slurries, polymer solutions and melts, surfactants, paints, foodstuffs, body fluids, soaps, inks, organic materials, adhesives, etc., is not adequately described by these models. Experiments on fluids that contain extremely small number of polymeric additives indicated that the skin friction near the rigid surface are about 30% to 50% lower than those without additives [1, 2]. Such discrepancies in experimental results and theoretical predictions have been attributed several factors, including surface effects and micro rotational effects of molecules. These experimental findings seem to imply that Navier- Stokes equations can no longer predict fluid behavior accurately in the above-mentioned situations. The inadequacy of the classical continuum theory has led to the development of theories of microcontinua in which continuous media possess not only mass and velocity but also substructure. Also, there exist different approaches to study the mechanics of fluids with a substructure. Several theories such as the theory of anisotropic fluids [3], the theory of simple micro fluids [4] and polar fluids [5] have been developed to take into account of geometry, deformation and intrinsic motion of individual material particles.

Within a family of micro fluids, there is a sub-class of fluids called the micropolar fluids [6]. The theory of micropolar fluids asserts that micropolar fluids can support couple stresses and body couples and exhibit micro rotational effects. In such fluids, rigid particles contained in a small volume element can rotate about the centroid of the volume element, in an average sense, described by an independent micro-rotation vector. The micropolar fluid model exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Physically, micropolar fluids may be described as non-Newtonian fluids consisting of dumb-bell molecules or short rigid cylindrical element, polymer fluids, fluid

suspension, etc. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. Micropolar fluids can also model anisotropic fluids, liquid crystals with rigid molecules, magnetic fluids, cloud with dust, muddy fluids and other biological fluids [7]. In the light of the theory, many classical flows have been re-examined to determine the effects of the fluid microstructure. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media are presented by Lukaszewicz [8]. Thus, micropolar fluid model is not a generalization for generalization sake of the Navier-Stokes model, but is a physically relevant model that has many applications.

The governing equations of convective heat and mass transfer in Newtonian or non-Newtonian fluids constitute a set of nonlinear nonhomogeneous differential equations for which closed-form solution cannot be obtained. Hence these equations have to be solved numerically. However no single numerical method is applicable to every non-linear differential equation. One of the popular methods those are available to solve these nonlinear differential equations Keller-box method [9]. This method has been proven to be adequate for boundary layer equations and is seen to give accurate results. This method is clearly discussed in the textbooks [10, 11]. The method has the following four main steps:

- Reduce the equation or system of equations to a first order system;
- Write the difference equations using central differences;
- Linearize the resulting algebraic equations (if they are non-linear) by Newtons method and write them in matrix-vector form;
- Use the block-tridiagonal-elimination technique to solve the linear system.

This method has been widely used and it seems to be the most flexible of the common methods. The scheme is also applicable to various types of boundary layer flow problems [12-16], which are the free and mixed convection flows.

Motivated by the earlier studies of researchers and in view of the significance of possible applications, the aim of this research is to study

some convection boundary layer flows in micropolar fluid in the presence of magnetic field thermal and solutal stratification, and cross-diffusion, mixed convection parameter effects. The problems considered deal with vertical plate geometry. Majority of the studies, reported in literature, on convective heat and mass transfer in micropolar fluid saturated porous medium deal with local similarity solutions and non-similarity solutions. But, in the present analysis, an attempt is made to obtain similarity solutions. The rest of the paper is organized as follows: mathematical formulation and method of solution with different boundary conditions are discussed in section 2 and 3. The boundary layer analysis for free convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with variable temperature and concentration wall conditions in the presence of a uniform magnetic field are discussed in section 4 followed by conclusions are discussed in section 5.

2. MATHEMATICAL FORMULATION

Consider a steady, laminar, incompressible, two-dimensional free convective heat and mass transfer along a semi-infinite vertical plate embedded in a doubly stratified, electrically conducting micro-polar fluid. Choose the coordinate system such that x axis is along the vertical plate and y axis normal to the plate. The physical model and coordinate system are shown in Fig. 1. The plate is maintained at temperature $T_w(x)$ and concentration $C_w(x)$. The temperature and the mass concentration of the ambient medium are assumed to be linearly stratified in the form $T_\infty(x) = T_\infty + A_1(x)$ and $C_\infty(x) = C_\infty + B_1(x)$, respectively, where A_1 and B_1 are constants and varied to alter the intensity of stratification in the medium and $T_\infty, 0$ and $C_\infty, 0$ are the beginning ambient temperature and concentration at $x = 0$, respectively. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field.

Using the Boussinesq and boundary layer approximations, the governing equations for the micro-polar fluid are given by

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \quad 2.1.1$$

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = \left(\frac{\mu + \kappa}{\rho}\right) \frac{\partial^2 \bar{u}}{\partial y^2} + \left(\frac{\kappa}{\rho}\right) \frac{\partial \bar{\omega}}{\partial y} + g^* [\beta_T (\bar{T} - \bar{T}_\infty)] + \beta_c (\bar{C} - \bar{C}_\infty) - \sigma \frac{B_0^2}{\rho} \bar{u} \quad 2.2.2$$

$$\bar{u} \frac{\partial \bar{\omega}}{\partial x} + \bar{v} \frac{\partial \bar{\omega}}{\partial y} = \left(\frac{\gamma}{\rho j}\right) \frac{\partial^2 \bar{\omega}}{\partial y^2} - \frac{\kappa}{\rho j} \left(2\bar{\omega} + \frac{\partial \bar{u}}{\partial y}\right) \quad 2.2.3$$

$$\bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} = \alpha \frac{\partial^2 \bar{T}}{\partial y^2} \quad 2.2.4$$

$$\bar{u} \frac{\partial \bar{C}}{\partial x} + \bar{v} \frac{\partial \bar{C}}{\partial y} = D \frac{\partial^2 \bar{C}}{\partial y^2} \quad 2.2.5$$

where u and v are the components of velocity along x and y directions, respectively, x is the component of micro rotation whose direction of rotation lies in the xy -plane, g^* is the gravitational acceleration, T is the temperature, C is the concentration, βT is the coefficient of thermal expansions, βc is the coefficient of solutal expansions, B_0 is the coefficient of the magnetic field, μ is the dynamic coefficient of viscosity of the fluid, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, ν is the kinematic viscosity, α is the thermal diffusivity, and D is the molecular diffusivity.

The boundary conditions are:

$$\bar{u} = 0, \bar{v} = -V_0, \bar{\omega} = 0, \bar{T} = T_w(\bar{x}), \bar{C} = C_w(\bar{x}) \text{ at } \bar{y} = 0 \\ \bar{u} \rightarrow 0, \bar{\omega} \rightarrow 0, \bar{T} \rightarrow T_\infty, \bar{C} \rightarrow C_\infty \text{ as } \bar{y} \rightarrow \infty$$

where the subscripts w and ∞ indicates the conditions at wall and at the outer edge of the boundary layer, respectively. The boundary condition $\bar{\omega} = 0$ in Eq. 2.2.3 at wall, i.e., at $\bar{y} = 0$, represents the case of concentrated particle flows in which the micro elements close to the wall are not able to rotate, due to the no-slip condition.

3. METHOD OF SOLUTION

Introducing the following non-dimensional variables to non-dimensionalize the governing equations

$$x = \frac{\bar{x}}{L}; y = \frac{\bar{y}}{L}; u = \frac{\bar{u}L}{\nu Gr^{1/2}}; v = \frac{\bar{v}L}{\nu Gr^{1/4}}; w = \frac{\bar{w}L}{\nu Gr^{3/4}} \\ Gr = \frac{g^* [\beta_T \delta T L^3]}{\nu^2}; \theta = \frac{\bar{T} - T_\infty}{T_w(x) - T_\infty}; \phi = \frac{\bar{C} - C_\infty}{C_w(x) - C_\infty}$$

We obtain the following equations which are dimensionless

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (1 + K) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial \bar{\omega}}{\partial y} + x(\theta + \mathfrak{B}\phi) - Mu \quad 3.1.1$$

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \left(1 + \frac{K}{2}\right) \frac{\partial^2 \omega}{\partial y^2} + \kappa \left(2\omega + \frac{\partial u}{\partial y}\right) \quad 3.1.2$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + \frac{u\theta}{x} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad 3.1.3$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} + \frac{u\phi}{x} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} \quad 3.1.4$$

where $K = \frac{\kappa}{\mu}$ is the Coupling number, $\mathfrak{B} = \frac{\beta_c \Delta C}{\beta_T \Delta T}$ is the buoyancy parameter, $M = \frac{\sigma B_0^2 L^3}{\rho \nu^2 Gr}$ is the magnetic parameter, $Pr = \frac{\theta}{\alpha}$ is the Prandtl number and $Sc = \frac{\nu}{D}$ is the Schmidt number

Introducing the following Lie Group transformations,

$$x = \hat{x} e^{-\epsilon \alpha_1}; y = \hat{y} e^{-\epsilon \alpha_2}; \psi = \hat{\psi} e^{-\epsilon \alpha_3}; \omega = \hat{\omega} e^{-\epsilon \alpha_4}; \theta = \hat{\theta} e^{-\epsilon \alpha_5}; \phi = \hat{\phi} e^{-\epsilon \alpha_6}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are transformation parameters and ϵ is a small parameter, this scaling group of transformations transform coordinates $(x, y, \psi, \omega, \theta, \phi)$ to $(\hat{x}, \hat{y}, \hat{\psi}, \hat{\omega}, \hat{\theta}, \hat{\phi})$ and above five equations and their boundary conditions are invariant under the point transformations, along with the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}; v = -\frac{\partial \psi}{\partial x}$$

so that continuity equation 2.1.1 is satisfied, then the equations 2.1.1 to 2.1.5 and boundary conditions are invariant under the point transformations, hence we obtain the following non-dimensional differential equations.

$$e^{\epsilon(\alpha_1 + 2\alpha_2 - 2\alpha_3)} \left[\frac{\partial \hat{\psi}}{\partial \hat{y}} \frac{\partial^2 \hat{\psi}}{\partial \hat{x} \partial \hat{y}} - \frac{\partial \hat{\psi}}{\partial \hat{x}} \frac{\partial^2 \hat{\psi}}{\partial \hat{y}^2} \right] = (1 + K) e^{\epsilon(3\alpha_2 - \alpha_3)} \frac{\partial^3 \hat{\psi}}{\partial \hat{y}^3} + K e^{\epsilon(\alpha_2 - \alpha_4)} \frac{\partial \hat{\omega}}{\partial \hat{y}} \\ + \hat{x} \left[\hat{\theta} e^{-\epsilon(\alpha_1 + \alpha_5)} + \beta \hat{\phi} e^{-\epsilon(\alpha_1 + \alpha_6)} \right] - M e^{\epsilon(\alpha_2 - \alpha_3)} \frac{\partial \hat{\psi}}{\partial \hat{y}} \quad 3.1.5$$

$$e^{\epsilon(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4)} \left(\frac{\partial \hat{\psi}}{\partial \hat{y}} \frac{\partial \hat{\omega}}{\partial \hat{x}} - \frac{\partial \hat{\psi}}{\partial \hat{x}} \frac{\partial \hat{\omega}}{\partial \hat{y}} \right) = \left(1 + \frac{\kappa}{2}\right) e^{\epsilon(2\alpha_2 - \alpha_4)} \frac{\partial^2 \hat{\omega}}{\partial \hat{y}^2} - K \left[2e^{-\epsilon \alpha_4} \hat{\omega} + e^{\epsilon(2\alpha_2 - \alpha_3)} \frac{\partial^2 \hat{\psi}}{\partial \hat{y}^2} \right] \quad 3.1.6$$

$$e^{\epsilon(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_5)} \left[\frac{\partial \hat{\psi}}{\partial \hat{y}} \frac{\partial \hat{\theta}}{\partial \hat{x}} - \frac{\partial \hat{\psi}}{\partial \hat{x}} \frac{\partial \hat{\theta}}{\partial \hat{y}} + \frac{\hat{\theta}}{\hat{x}} \frac{\partial \hat{\psi}}{\partial \hat{y}} \right] = e^{\epsilon(2\alpha_2 - \alpha_5)} \frac{1}{Pr} \left[\frac{\partial^2 \hat{\theta}}{\partial \hat{y}^2} \right] \quad 3.1.7$$

$$e^{\epsilon(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_6)} \left[\frac{\partial \hat{\psi}}{\partial \hat{y}} \frac{\partial \hat{\phi}}{\partial \hat{x}} - \frac{\partial \hat{\psi}}{\partial \hat{x}} \frac{\partial \hat{\phi}}{\partial \hat{y}} + \frac{\hat{\phi}}{\hat{x}} \frac{\partial \hat{\psi}}{\partial \hat{y}} \right] = e^{\epsilon(2\alpha_2 - \alpha_6)} \frac{1}{Sc} \left[\frac{\partial^2 \hat{\phi}}{\partial \hat{y}^2} \right] \quad 3.1.8$$

along with the transformed boundary conditions

$$\left. \begin{aligned} e^{\epsilon(\alpha_2 - \alpha_3)} \frac{\partial \hat{\psi}}{\partial \hat{y}} = 0, e^{\epsilon(\alpha_1 - \alpha_3)} \frac{\partial \hat{\psi}}{\partial \hat{x}} = -V_0 \sqrt{Re}, e^{-\epsilon \alpha_4} \hat{\omega} = 0, e^{-\epsilon \alpha_5} \hat{\theta} = l \text{ at } e^{-\epsilon \alpha_2} \hat{y} = 0 \\ e^{\epsilon(\alpha_2 - \alpha_3)} \frac{\partial \hat{\psi}}{\partial \hat{y}} \rightarrow 0, e^{-\epsilon \alpha_4} \hat{\omega} \rightarrow 0, e^{-\epsilon \alpha_5} \hat{\theta} \rightarrow l - \epsilon_1, e^{-\epsilon \alpha_6} \hat{\phi} \rightarrow l - \epsilon_2 \text{ as } e^{-\epsilon \alpha_2} \hat{y} \rightarrow \infty \end{aligned} \right\}$$

Since the group transformations keeps the system invariant, We then have following relationships among the parameters

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 = \alpha_2 - \alpha_4 = \alpha_1 + \alpha_5 = \alpha_1 + \alpha_6 \\ = \alpha_2 - \alpha_3 \\ \alpha_2 - \alpha_3 = \alpha_1 - \alpha_4 = \alpha_1 - \alpha_3 = \alpha_2 - \alpha_4 = 2\alpha_2 - \alpha_4 = \alpha_4 \\ = 2\alpha_2 - \alpha_3$$

Solving the linear system of equations, we have the following relationship among the exponents

$$\alpha_1 = \alpha_3 = \alpha_4 \\ \alpha_2 = \alpha_5 = \alpha_6 = 0$$

So the infinitesimal transformations reduces to point transformations in one parameter as follows

$$x = \hat{x}e^{-\varepsilon\alpha_1}; y = \hat{y}; \psi = \hat{\psi}e^{-\varepsilon\alpha_1}; \omega = \hat{\omega}e^{-\varepsilon\alpha_1}; \theta = \hat{\theta}; \phi = \hat{\phi} \quad 3.1.9$$

After expanding the Lie group of point transformations in one parameter using the Taylor's series in powers of ε by considering the terms up to $O(\varepsilon)$, we get

$$\hat{x} - x = x\varepsilon\alpha_1, \hat{y} - y = 0, \hat{\psi} - \psi = \psi\varepsilon\alpha_1, \hat{\omega} - \omega = \omega\varepsilon\alpha_1, \hat{\theta} - \theta = 0, \hat{\phi} - \phi = 0 \quad 3.1.10$$

The corresponding characteristic equations are given by

$$\frac{dx}{x\alpha_1} = \frac{dy}{0} = \frac{d\psi}{\psi\alpha_1} = \frac{d\omega}{\omega\alpha_1} = \frac{d\theta}{0} = \frac{d\phi}{0} \quad 3.1.11$$

The self-similar solutions of characteristic equations give the similarity transformations as below

$$\hat{y} = \eta, \hat{\psi} = \hat{x}f(\eta), \hat{\omega} = \hat{x}g(\eta), \hat{\theta} = \theta(\eta), \hat{\phi} = \phi(\eta) \quad 3.1.12$$

Hence, the transformed governing equations are

$$(f')^2 - ff'' = (1 + K)f''' + Kg' + \theta + \mathfrak{B}\phi - Mf' \quad 3.1.13$$

$$f'g - fg' = \left(1 + \frac{k}{2}\right)g'' - 2Kg - Kf'' \quad 3.1.14$$

$$\frac{1}{Pr}\theta'' - \theta f' + f\theta' - \varepsilon_1 f' = 0 \quad 3.1.15$$

$$\frac{1}{Sc}\phi'' - \phi f' + f\phi' - \varepsilon_2 f' = 0 \quad 3.1.16$$

The boundary conditions associated with these equations are

$$\left. \begin{aligned} f' &= 0, f = -f_0, g = 0, \theta = 1 - \varepsilon_1, \phi = 1 - \varepsilon_2 \quad \text{at } \eta = 0 \\ f' &\rightarrow 0, g \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \right\}$$

4. RESULTS AND DISCUSSION

The non-linear non-homogeneous ordinary differential equations are solved numerically using the Keller-box implicit method [11]. This method has a second order accuracy and unconditionally stable. The calculations are repeated until a suitably chosen convergence is satisfied and the calculations are stopped when $\delta f'(0) \leq 10^{-8}$, $\delta \theta'(0) \leq 10^{-8}$ and $\delta \phi'(0) \leq 10^{-8}$. In order to see the effects of step size ($\Delta\eta$) we executed the code for our model with three different step sizes as $\Delta\eta = 0.001$, $\Delta\eta = 0.01$ and $\Delta\eta = 0.05$ and in each case we found very good agreement between them on different profiles. After some trials we imposed a maximal value of h (i.e., η value) as 6 and a grid size of $\Delta\eta$ as 0.01. In order to study the effects of the coupling number K , magnetic field parameter M , suction/injection

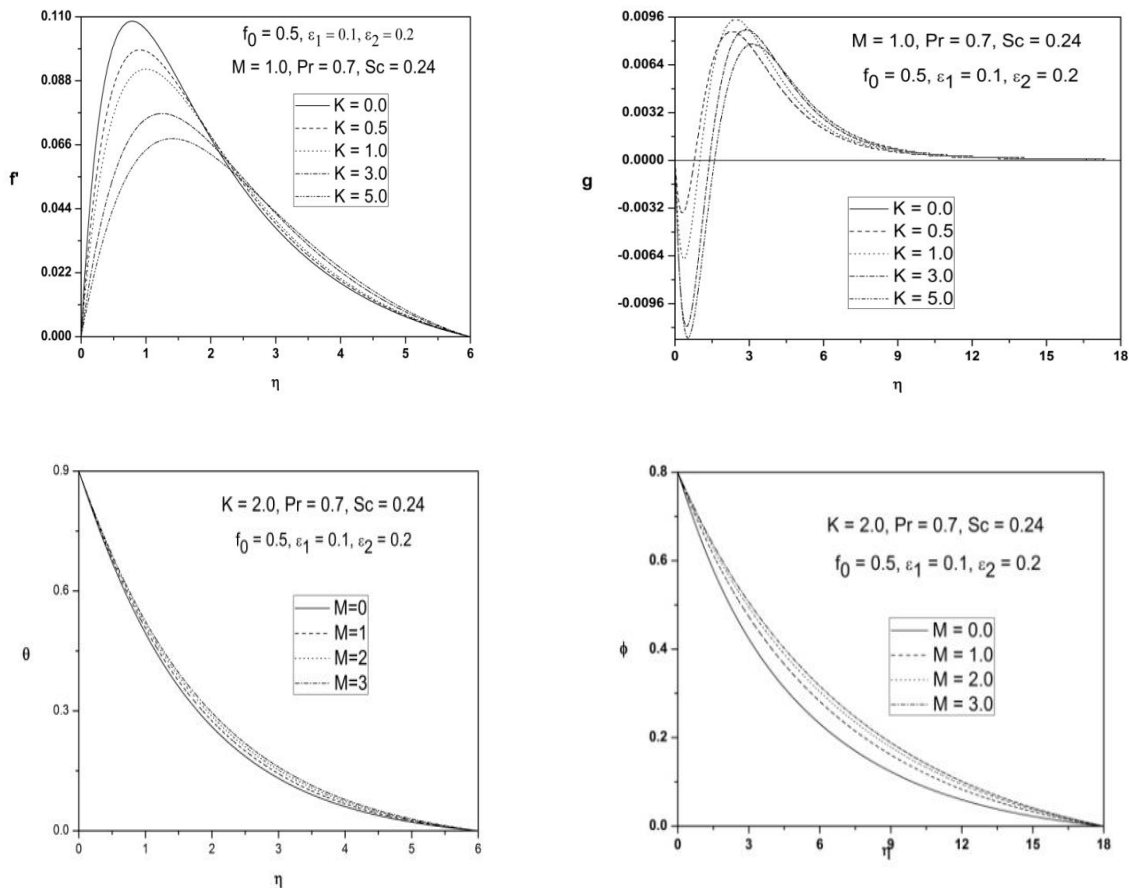


Figure 1: (a) Velocity, (b) Microrotation, (c) Temperature and (d) Concentration Profiles for various values of Coupling parameter K

parameter f_0 , thermal stratification parameter ε_1 and solutal stratification parameter ε_2 on the physical quantities of the flow.

Figure 1 depicts the variation of coupling number (K) on the profiles of velocity, micro rotation, temperature and concentration with η . The coupling number K characterizes the coupling of linear and rotational motion arising from the micro motion of the fluid molecules. Hence, K signifies the coupling between the Newtonian and rotational viscosities. As K increases, the effect of microstructure becomes significant, whereas with a small value of K the individuality of the substructure is much less pronounced. As $K \rightarrow 0$ i.e. $K \rightarrow 0$, the micro polarity is lost and the fluid behaves as nonpolar fluid and thus the case of $K \rightarrow 0$ corresponds to the case of viscous fluid. It is observed from Figure 1(a) that the velocity

decreases with the increase of K . The maximum of velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall with an increase of K . The velocity in case of micropolar fluid is less than that in the viscous fluid case ($K \rightarrow 0$ corresponds to viscous fluid). It is seen from Fig.1(b) that the microrotation component decreases near the vertical plate and increases far away from the plate with increasing coupling number K . The microrotation tends to zero as $K \rightarrow 0$ as is expected that in the limit $K \rightarrow 0$, i.e. $K \rightarrow 0$ the Eqs. (1) and (2) are uncoupled with Eq. (3) and they reduce to viscous fluid flow equations. It is noticed from Figure 2(a) that the temperature increases with increasing values of coupling number. It is clear from Figure 2(b) that the non-dimensional concentration increases from Newtonian case to non-Newtonian case (with increasing values of K).

The variation of the non-dimensional velocity, microrotation, temperature and concentration profiles with η for different values of magnetic parameter is illustrated in Figure 3. It is observed from Figure 6 that

velocity decreases as the magnetic parameter (M) increase

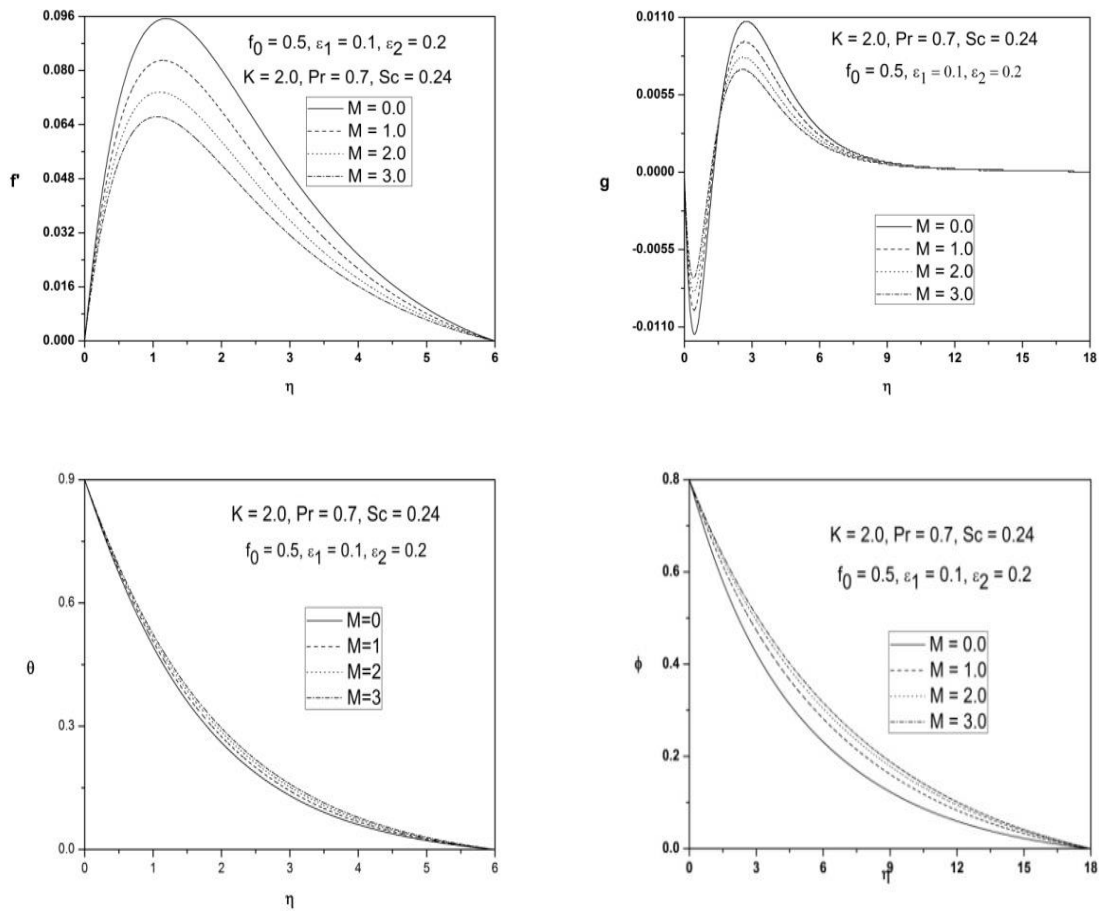
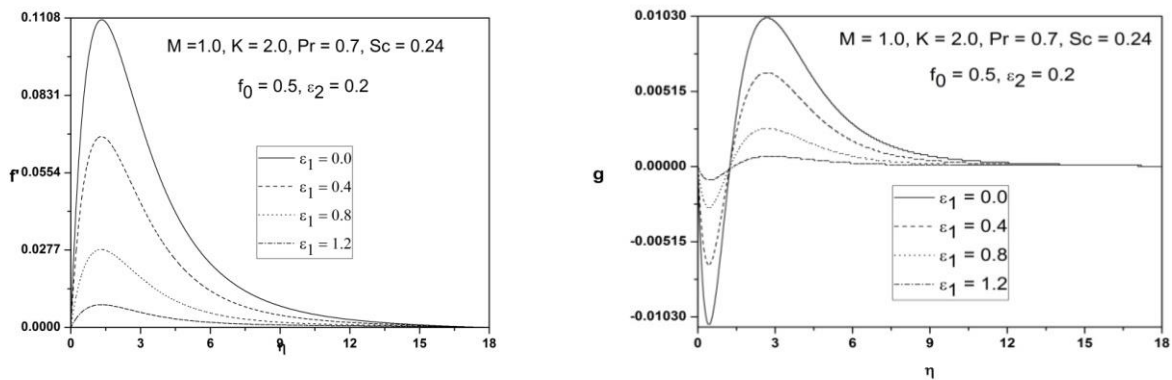


Figure 2: (a) Velocity, (b) Microrotation, (c) Temperature and (d) Concentration Profiles for various values of magnetic parameter M

From Figure 7, it is clear that the microrotation component increases near the plate and decreases far away from the plate for increasing values of M . It is noticed from Figure 8 that the non-dimensional fluid temperature increases with increasing values of magnetic parameter. It is clear from Fig. 9 that the non-dimensional fluid concentration increases with increasing values of M . Application of a uniform magnetic field normal to

the flow direction produces a force which acts in the negative direction of flow. This force is called the Lorentz force which tends to slow down the movement of the electrically conducting fluid in the vertical direction. This retardation effect is accompanied by an appreciable increase in the fluid temperature and concentration. These behaviors are clearly depicted in Figures 6 to 9.



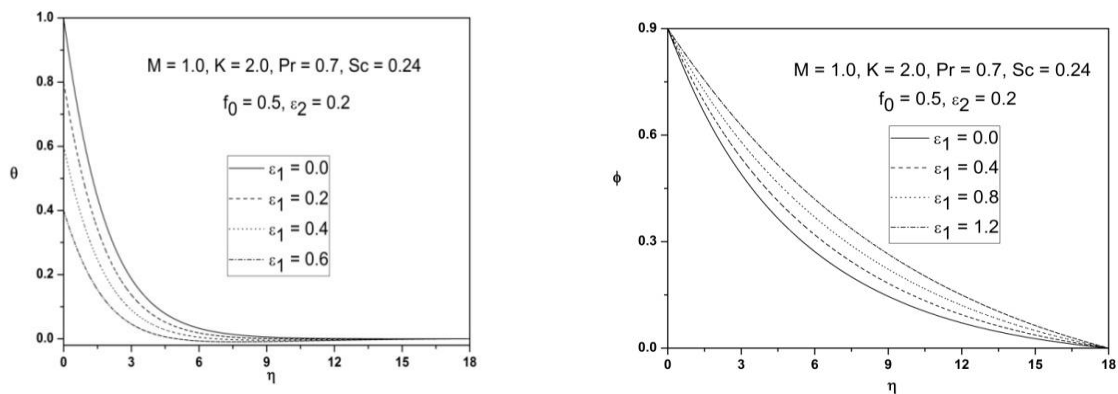


Figure 3: (a) Velocity, (b) Microrotation, (c) Temperature and (d) Concentration Profiles for various values of thermal stratification parameter ε_1

The effect of thermal stratification parameter ε_1 on the non-dimensional velocity, microrotation, temperature and concentration is shown in Figures from 10 – 13. It is observed from Fig. 10 that the velocity decreases with the increase of thermal stratification ε_1 . This is because thermal stratification reduces the effective convective potential between the heated plate and the ambient fluid in the medium. Hence, the thermal stratification effect reduces the velocity in the boundary layer. From Figure 11, we observe that the values of microrotation change sign from negative to positive at the critical point $\eta = 0.8917$ within the boundary layer. Also, it is clear that the magnitude of the microrotation increases with an increase in thermal stratification parameter. It is noticed from Figure 12 that the non-dimensional temperature of the fluid decreases with the increase of thermal stratification parameter. When the thermal

stratification effect is taken into consideration, the effective temperature difference between the plate and the ambient fluid will decrease; therefore, the thermal boundary layer is thickened and the temperature is reduced. Figure 13 demonstrates that the concentration of the fluid increases with the increase of thermal stratification parameter. It can be noted that the effect of the stratification on temperature is the formation of a region with a temperature deficit (i.e., a negative dimensionless temperature). This is in tune with the observation made in references [Gebhart et al. (1988), Prandtl (1952), Murthy et al. (2004), Lakshmi Narayana and Murthy Narayana and Murthy (2006a,b) and Jaluria and Himasekhar (1983)].

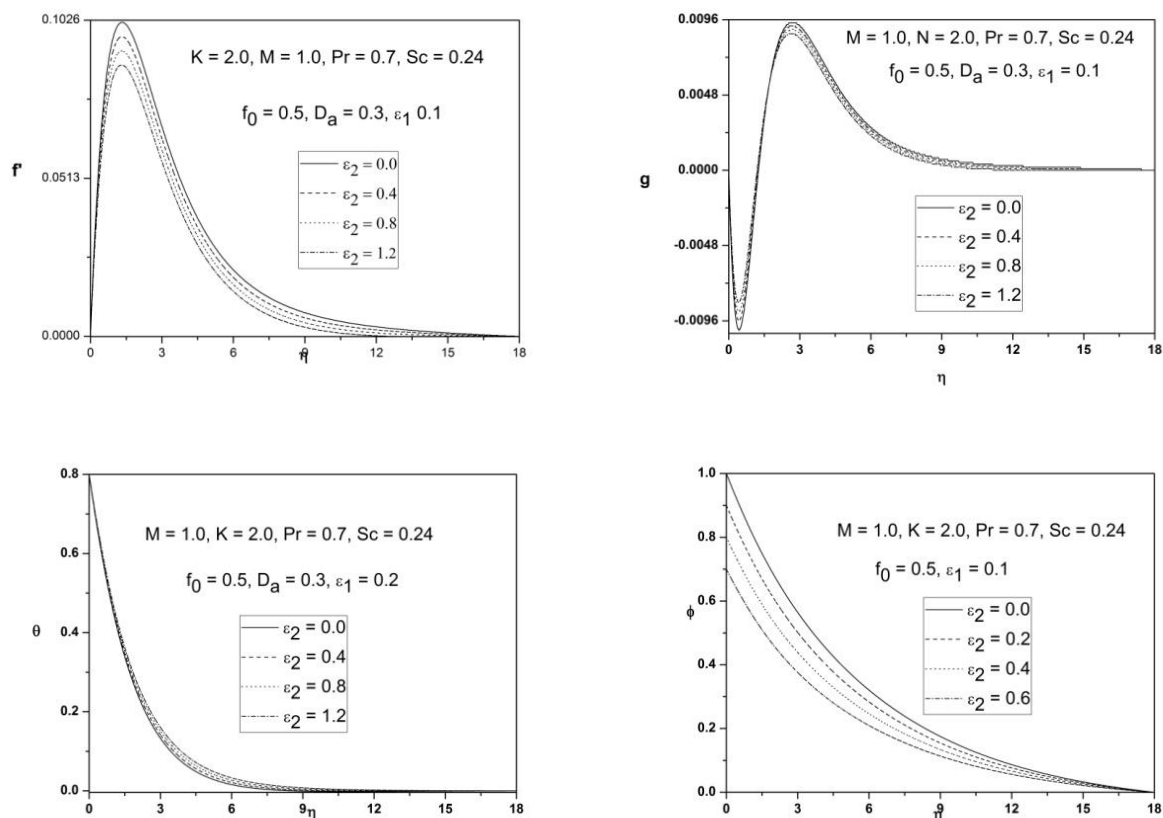


Figure 4: (a) Velocity, (b) Microrotation, (c) Temperature and (d) Concentration Profiles for various values of thermal stratification parameter ε_2

The dimensionless velocity component, microrotation, temperature and concentration for different values of solutal stratification parameter ε_2 is depicted in Fig. from 14 - 17. From Fig. 14 it is observed that the velocity of the fluid decreases with the increase of solutal stratification parameter. From Figure 15 we observe that the microrotation values change sign from negative to positive at the critical point $\eta = 0.88$ within the boundary layer. Also, it is clear that the magnitude of the microrotation increases with an

increase in solutal stratification parameter. It is noticed from Fig. 16 the temperature of the fluid increases with the increase of solutal stratification parameter. It is clear from Figure 17 that the non-dimensional concentration of the fluid decreases with the increase of thermal stratification parameter.

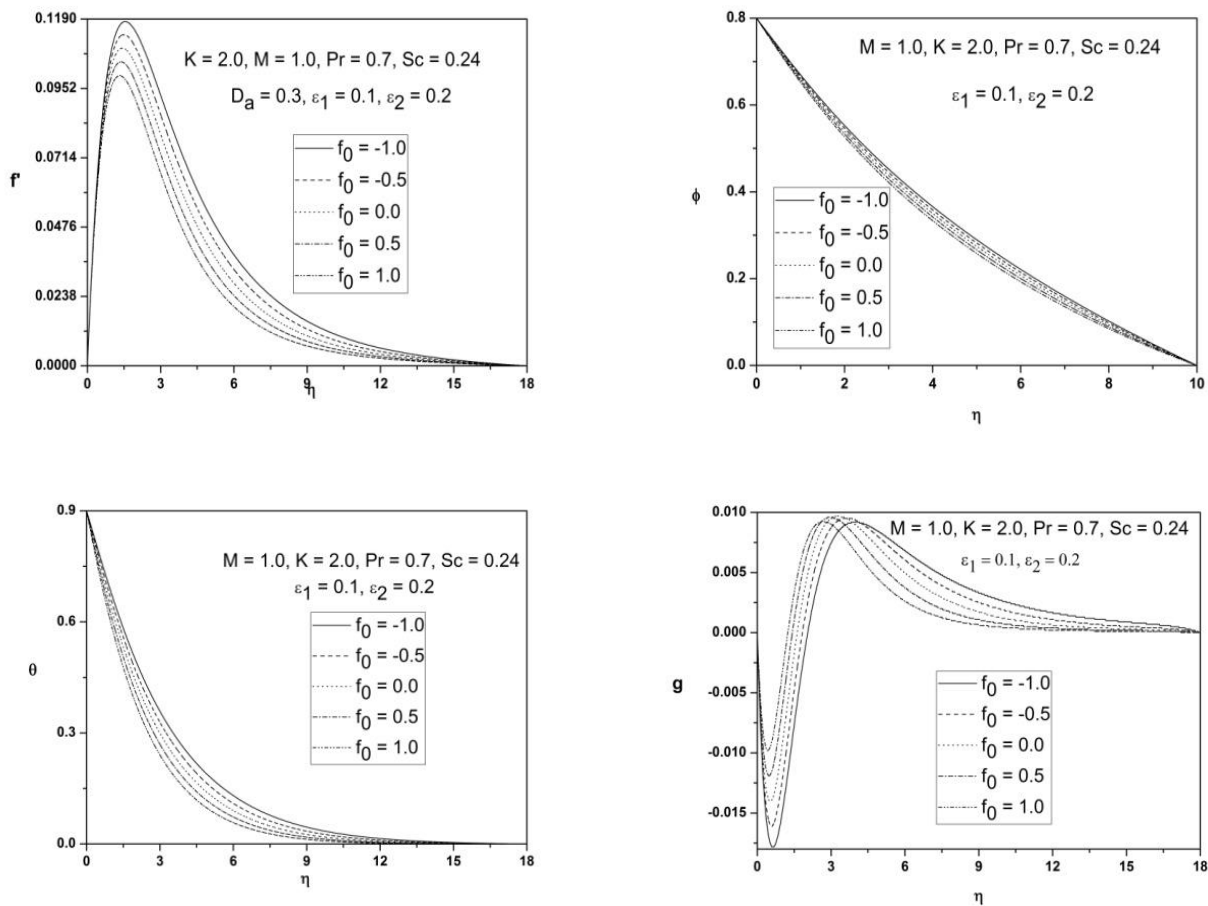


Figure 5: (a) Velocity, (b) Microrotation, (c) Temperature and (d) Concentration Profiles for various values of suction/injection

5. CONCLUSION

In the present analysis, an attempt is made to obtain the boundary layer analysis for free convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with variable temperature and concentration wall conditions in the presence of a uniform magnetic field

- In this paper, a boundary layer analysis for free convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with variable temperature and concentration wall conditions in the presence of a uniform magnetic field of magnitude B_0 , Thermal and Solutal stratification effects is considered.
- Using the similarity variables, the governing equations are transformed into a set of parabolic equations and numerical solution for these equations has been presented for different values of parameters.
- The higher values of the coupling number K (i.e., the effect of microrotation becomes significant) result in lower velocity distribution but higher wall temperature; wall concentration distributions in the boundary layer compared to the Newtonian fluid case.
- The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfers in the micropolar fluid is lower compared to that of the Newtonian fluid.
- The present analysis has also shown that the flow field is appreciably influenced by the Magnetic parameter (M), Thermal stratification parameter, and Solutal stratification parameter.

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